Unemployment and Inflation Regimes

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Abstract: In this paper we study 2-state Markov switching VAR models of monthly unemployment and inflation for three countries: Sweden, United Kingdom, and the United States. The primary purpose is to examine if periods of low inflation are associated with high or low unemployment volatility. We find that MS-VAR models seem to provide a better description of the data than single regime VARs and need fewer lags to account for serial correlation. To interpret the regimes the empirical results are compared with the predictions from a version of Rogoff's (1985) model of monetary policy. We find that both the theoretical and the empirical results suggest that an increase in central bank "conservativeness" can be associated with either a higher or a lower variance in unemployment. In the U. S. case we find that the variance of unemployment is lower in the low inflation regime than in the high inflation regime, while the Swedish and the U. K. cases suggest that unemployment variability is higher in the low inflation regime.

Keywords: Cointegration, monetary policy, Phillips curve, regime switching, unemployment volatility.

JEL Classification Numbers: C32, E31, E52.

1. Introduction

This paper tries to shed light on the following question: Is the volatility of unemployment higher when inflation is low than when it is high? This question is interesting because many countries seem to have made switches to low-inflation regimes during the past two decades. Moreover, there are theoretical arguments which suggest that higher ambitions to fight inflation may be associated with increased volatility in real economic activity; see, e.g., Rogoff (1985) and Taylor (1979, 1994).

It is well known, through both theoretical and empirical studies, that the relation between inflation and unemployment depends on monetary policy.[1] Most empirical work has, however, concentrated on the relation between the *levels* of the two variables (e.g. on whether a "long run Phillips curve" has a positive or a negative slope). Some studies have examined if

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[1] For instance, Persson and Tabellini (1993), Walsh (1995), and Svensson (1997) have shown that, in principle, monetary policy may be designed so that the trade-off between the level of inflation and real volatility can be avoided. Ireland (1999) and Haldane and Quah (1999) provide both theoretical and empirical arguments on the nature of the inflation-unemployment relation in the U. S. and the U. K., respectively.

a relation between the mean of inflation and the variance of unemployment can be found in cross-country data. Alesina and Summers (1993) and Jonsson (1995) have failed to detect such a relation.[2]

The use of cross-country data is based on the assumption that inflation-unemployment relations are constant over time within individual countries, and that differences between countries are large enough to make cross-country comparisons a meaningful way to examine the relationship. There do indeed seem to exist differences between countries in terms of monetary policy regimes, e.g. regarding the degree of central bank independence, that are quite persistent (see, for instance, Alesina, 1988, and Cukierman, 1992). But it is also the case that there have been changes in monetary policy within countries, due to, among other things, policy makers' preferences (or beliefs) regarding the trade-off between inflation and unemployment. If so, important time series information is neglected in cross-country data.

These arguments suggest that it may be worthwhile to take a closer look at the problem we are interested in using time series from different countries. They also naturally lead us to examine the relevance of a nonlinear econometric model: We want to allow for the possibility that the (conditional) variance of unemployment varies over time in a way that is related to movements in average inflation.[3]

In this paper we look at unemployment and inflation data from three countries: the U. S., the U. K., and Sweden. Following King and Watson (1994) and Ireland (1999) we first estimate, for each country, bivariate vector autoregressive (VAR) models and test if there is a long run (cointegration) relation between inflation and unemployment. We also examine if the bivariate VAR models appear to be well specified or if it is possible to detect regime changes. Specifically, two-state Markov switching vector autoregressive (MS-VAR) models are analysed using techniques suggested by Hamilton (1990, 1994, 1996), Timmermann (2000), and Warne (1999b).[4]

In order to interpret the empirical results we compare them with predictions from a version of Rogoff's (1985) model of monetary policy. We do not identify the parameters of this theoretical model in our empirical setting, but the parameters make it easier to discuss possible sources of regime shifts in the links between inflation and unemployment.

The paper is organized as follows: In Section 2 we give an overview of the MS-VAR model and its relations to our version of the Rogoff (1985) model; the derivation of the latter model

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[2] This result is *not* surprising, given the theoretical analyses mentioned in footnote 1.

[3] Nonlinear models have previously been applied in contexts similar to ours. Lee (1999) studies a bivariate GARCH model on U. S. data to examine the hypothesis of a relation between the variances of inflation and output. Eliasson (1999) tests for nonlinearity of a "short run Phillips curve" in Australia, Sweden and the U. K. Analyses by Gruen, Pagan, and Thompson (1999) and Sims (1999) are similar to ours and are further discussed in Section 5 below.

[4] See also Warne (1999a) and Jacobson, Lindh, and Warne (1998) for further presentation of the econometric model and other applications.

is presented in the Appendix. Section 3 contains both a description of the econometric models we apply and a rather detailed discussion of the results for the U. S. data. The evidence for the U. K. and Sweden are more briefly presented in Section 4. In Section 5 we discuss the relations between our analyses and other recent studies, in particular of the empirical properties of Phillips curves. Section 6 contains summary and conclusions.

2. Simple Models of Unemployment and Inflation

2.1. *Time Series Models*

In order to analyze the empirical relations between inflation (π_t) and unemployment (u_t) within a model which is quite simple, yet able to capture important stylized facts, King and Watson (1994), among others, use a VAR model:

$$x_t = \delta + \sum_{j=1}^{k} A_j x_{t-j} + \varepsilon_t, \tag{1}$$

where $x_t = (\pi_t, u_t)$, k is the lag length, and ε_t is a white noise residual with zero mean and covariance matrix Ω.

When we suspect that for instance monetary policy is subject to regime shifts over the sample, then a critical assumption in (1) is that the parameters are constant. In his study of quarterly U. S. data, Ireland (1999) does not find any signs of parameter nonconstancy in his VARMA model, although such evidence has earlier been reported for monthly U. S. data by King and Watson (1994) and recently by Stock and Watson (1999). Moreover, the results from Lee's (1999) multivariate GARCH study also suggest that the parameters are time varying.

If the parameters in a VAR model vary over time in a stationary fashion, due to e.g. recurring shifts in labor market conditions or monetary policy, then standard parameter constancy tests (such as the Chow test) may not be suitable for detecting such variations in the data. Instead, the shifts are likely to result in an empirical VAR model with serially correlated residuals (or a very long lag length) and conditional heteroskedasticity. In this paper we shall therefore analyze both single regime VAR models like (1) and regime dependent VAR models like

$$x_t = \delta_{s_t} + \sum_{j=1}^{k} A_{j,s_t} x_{t-j} + \varepsilon_t, \tag{2}$$

where s_t denotes an unobservable (discrete) regime variable which takes on a finite number of positive integer values $\{1, \dots, q\}$, and $\varepsilon_t | s_t \sim N(0, \Omega_{s_t})$. For simplicity we assume that s_t follows an ergodic Markov process with transition probabilities $\Pr[s_t = j | s_{t-1} = i, s_{t-2} = h, \dots, x_{t-1}, x_{t-2}, \dots] = p_{ij}$. The model in (2) is henceforth called a MS-VAR model.

The Markov assumption is in some situations questionable. For example, suppose s_t represents two monetary policy regimes. We would then typically expect these regimes to depend on, among other things, the recent development of inflation and unemployment. This could be modeled, as in Gray (1996), by letting the transition probabilities be time varying and depend on these variables. With the Markov assumption the regime process is exogenously determined, but as long as it is serially correlated (the transition probability $p_{ij} \neq \varpi_j$, the ergodic probability, for some i, j) then the current regime can be predicted using *only* past values of inflation and unemployment.

Still, we are interested in estimating the first two moments of x_t (or a stationary transformation of this vector) conditional on s_t. This will allow us to address the question if the volatility in unemployment is higher when inflation is low than when it is high. We know from e.g. Timmermann (2000) and Warne (1999b) how to compute these moments under the Markov assumption, but not for more complex regime processes.

In the next subsection we shall study a version of a simple and well known model of monetary policy. The purpose of that exercise is to obtain predictions about the behavior of inflation and unemployment in terms of possible sources for nonstationarity, a cointegration relationship, and how the first two moments of (a stationary transformation of) these variables behave when the parameters of the theoretical model change. The latter analysis helps us interpret possible sources of regime shifts in our data.

2.2. *A Version of Rogoff's Model*

An extension of the model by Rogoff (1985) can be used to derive the following relation between unemployment and unexpected changes in the price level:

$$u_t = n^s - n^u_{t-1} + \omega\alpha(n^d - n^u_{t-1}) + \left[\omega + \frac{1}{\alpha}\right](E_{t-1}p_t - p_t) + \omega\rho z_{t-1} - \frac{1}{\alpha}\varepsilon_{z,t}, \tag{3}$$

where p is the log of the price level and z is technology (the Solow residual). The latter is assumed to follow a first order autoregressive process with ρ being the autocorrelation and $\varepsilon_{z,t}$ being a mean zero technology shock having variance σ_z^2. The rate of unemployment increases if there is an increase in the intercept in the labor supply function, n^s, but it also goes up if there is an increase in the intercept in the labor demand function, n^d. The reason is that the period t nominal wage is determined in period $t-1$ and that wage setters (e.g., labor unions) choose a higher expected real wage if labor demand is expected to rise. This effect dominates the initial demand effect and, hence, unemployment rises. Wage setters want to stabilize employment around some desired level, n^u_{t-1}, and an increase in that level leads to lower wages and lower unemployment. The parameter α is capital's share of value added and it is equal to the inverse of the slope of the labor demand function; the higher is α, the less does the real wage affect labor demand and therefore unemployment. For the same reason, the higher is α, the less sensitive is labor demand and unemployment to (contemporaneous) technology shocks. The slope of the labor supply function is measured by ω and hence the (absolute value of the) elasticity of unemployment with respect to inflation surprises increases in ω.[5]

The central bank wants to stabilize inflation around the inflation target π^* and employment around the target n_t^*. The relative weight on inflation in the central bank's loss function is λ. The central bank takes the unemployment equation (3), wages, and inflation expectations as given, but the wage setters have rational expectations about monetary policy. The equilibrium inflation rate $\pi_t = p_t - p_{t-1}$ can be characterized as follows:

$$\pi_t = \pi^* + \frac{1}{\alpha\lambda(\lambda + \alpha^{-2})}\left(\lambda n_t^* + \frac{1}{\alpha^2}E_{t-1}n_t^*\right) - \frac{1}{\alpha\lambda}n_{t-1}^u - \frac{1}{1+\alpha^2\lambda}\varepsilon_{z,t}. \qquad (4)$$

There is a positive inflation bias, as in Barro and Gordon (1983) and Rogoff (1985), when the central bank strives for a higher employment rate than wage setters, i.e. if $n_t^*, E_{t-1}n_t^* > n_{t-1}^u$. Typically, the bias will be lower the higher is the weight on inflation in the central bank's objective function. Using (4) in (3) gives the following expression for unemployment in equilibrium:

$$u_t = n^s - n_{t-1}^u + \omega\alpha(n^d - n_{t-1}^u) - \frac{1+\alpha\omega}{1+\alpha^2\lambda}(n_t^* - E_{t-1}n_t^*) + \omega\rho z_{t-1} + \frac{\omega - \alpha\lambda}{1+\alpha^2\lambda}\varepsilon_{z,t}. \qquad (5)$$

It is noteworthy that the central bank's employment target only affects unemployment through surprises. If the target is higher than the expected, then the real wage is lower than the expected thereby having a positive demand and a negative supply effect on employment.

2.3. *Time Series Properties of the Economic Model*

In contrast to the original model by Rogoff (1985), the version presented in this paper allows inflation and unemployment to be autocorrelated. Depending on the processes that govern technology and wage setters' and the central bank's employment targets, inflation and unemployment may even follow separate or common stochastic trends. These are desirable features of a model of these variables, given what we know about their empirical regularities.

The observed persistence in inflation is sometimes attributed to price stickiness and adaptive expectations (see e.g. Galí and Gertler, 1999). An additional source of persistence in inflation might be that monetary policy affects aggregate demand and output with a lag.

[5] The version of Rogoff's (1985) model that produces equations (3)–(5) is presented in the Appendix. In Rogoff's original model, wages are set so as to stabilize employment around the equilibrium level that would result if wages were perfectly flexible. This implies that the average rate of unemployment is zero, which is not desirable for our purposes.

It seems also likely that inflation persistence partly depends on central banks' preferences for employment stabilization and interest rate smoothing (see e.g. Svensson, 1999).

In principle, the observed persistence of unemployment could also be due to nominal rigidities and monetary policy. It seems at least equally plausible, however, that unemployment persistence depends on properties of the labor supply and the wage setting functions, i.e. some kind of real rigidities.[6] In our model, persistence in inflation is either due to persistence in wage setters' employment target or to persistence in the central banks' employment target. The former can also lead to unemployment persistence but not the latter. Persistence in technology, on the other hand, can only result in unemployment persistence.

Accordingly, from equation (4) it can be seen that inflation can only be nonstationary (integrated of order 1) if there is a stochastic trend (unit root) in either the central bank's or wage setters' employment target. Similarly, from equation (5) we find that unemployment is nonstationary if either there is a stochastic trend in wage setters' employment target or in technology ($\rho = 1$). Hence, a necessary condition for inflation and unemployment to be both nonstationary and cointegrated (there exists a linear combination which is stationary) is that there is a stochastic trend in wage setters' employment target. For sufficiency, it is also required that technology is stationary and that there is not a unique trend in the central bank's employment target.[7] This result is consistent with Ireland's model, where nonstationarity is due to a stochastic trend in the natural rate of unemployment.

Let us therefore first suppose that the employment targets evolve according to the stochastic process

$$n_t^u = n_{t-1}^u + \varepsilon_{u,t}, \tag{6}$$

$$n_t^* = n^* + \gamma n_{t-1}^u, \tag{7}$$

where $\varepsilon_{u,t}$ is white noise with zero mean, variance σ_u^2, and uncorrelated with $\varepsilon_{z,t}$. In addition suppose that technology is stationary with, for simplicity, $\rho = 0$. From equations (4) and (5) we then find that the linear combination $\pi_t - \beta_u u_t$ is stationary with

$$\beta_u = \frac{1-\gamma}{\alpha\lambda(1+\alpha\omega)}. \tag{8}$$

Hence, the cointegration relation between inflation and unemployment has a positive (negative) slope when the central bank's employment target is less (more) ambitious (abstracting from the constant n^*) than the wage setters' target, i.e. when γ is less (greater) than unity.

[6] Theoretical and empirical models of this issue have been presented by Jacobson, Vredin, and Warne (1997), Hansen and Warne (1997), and Ireland (1999), among others. Galí and Gertler (1999) suggest that real rigidities may also give rise to inflation persistence.

[7] This is based on the assumption that the employment targets do not depend on technology. If they do, then the model is consistent with e.g. a stationary unemployment rate and nonstationary technology when $n_t^u = (\omega/(1+\omega\alpha))z_t$ and $\rho = 1$.

From equation (8) it can also be seen that inflation is stationary when $\gamma = 1$, i.e. when the difference between the central bank's and wage setters' employment targets is constant. There is also a second case when inflation is stationary while unemployment is not. From equations (4) and (5) we have already noted that although the contemporaneous technology shock influences inflation, past technology shocks affect only unemployment (via the labor supply function). Let us therefore consider a second case when, for simplicity, both employment targets are constant, $n_t^* = n^*$ and $n_t^u = n^u$, while technology follows a random walk ($\rho = 1$). We now find that inflation is given by

$$\pi_t = \pi^* + \frac{1}{\alpha\lambda}\left(n^* - n^u\right) - \frac{1}{1 + \alpha^2\lambda}\varepsilon_{z,t}. \tag{9}$$

Hence, there is a positive (negative) inflation bias when n^* is greater (less) than n^u. Moreover, unemployment is now given by

$$u_t = n^s - n^u + \omega\alpha(n^d - n^u) + \omega z_{t-1} + \frac{\omega - \alpha\lambda}{1 + \alpha^2\lambda}\varepsilon_{z,t}. \tag{10}$$

The contemporaneous effect on inflation from a technology shock is negative, while the effect on unemployment is ambiguous. In the long run, however, inflation is not affected while unemployment increases since labor demand is unaffected (the real wage increases one-for-one with the technology shock) and labor supply rises.

From an empirical perspective, these results mean that if inflation is stationary and unemployment nonstationary, then this information cannot help us distinguish between a stochastic trend in the employment targets and a stochastic trend in technology. Should, however, $\beta_u \neq 0$ (as suggested by Ireland, 1999) then the model predicts that this is due to a common trend in the employment targets (or a stochastic trend in the natural rate of unemployment).

2.4. *Regime Shift Predictions from the Economic Model*

The parameters of the model in (4) and (5) are natural candidates for explaining regime shifts. Since the first two moments of (π_t, u_t) do not exist when the process is nonstationary, let us first consider the case when both inflation and unemployment are stationary. Suppose, again for simplicity, that the employment targets are constant ($n_t^* = n^*, n_t^u = n^u$) and that technology is white noise ($\rho = 0$). We now have that, in equilibrium, inflation is given by equation (9), while unemployment is

$$u_t = n^s - n^u + \omega\alpha(n^d - n^u) + \frac{\omega - \alpha\lambda}{1 + \alpha^2\lambda}\varepsilon_{z,t}. \tag{11}$$

In Table 1 we summarize how the means and variances of inflation and unemployment are affected by changes in the parameters of the model for this particular case.

It is noteworthy that this model does *not* predict that the variance of unemployment is unambiguously increasing in the central bank's inflation aversion parameter, λ. A low λ implies that technology shocks are allowed to have a large effect on the price level. This, in turn, implies that the real wage responds strongly to such shocks, which partly offsets the direct effects of such shocks on labor demand. In the extreme case of $\lambda = 0$, monetary policy stabilizes employment at n^u, but this does not stabilize unemployment, which is also affected by the effects of price surprises on labor supply.[8] At low values of $\lambda (< \omega/\alpha)$, an increase in the central bank's inflation aversion may thus lower the variance of unemployment.[9]

The effects in Table 1 from the monetary policy parameters may loosely be interpreted as the model's predictions of the effects from monetary policy regime shifts. However, these predictions are based on the assumption that inflation and unemployment are both stationary. In Table 2 we have therefore summarized how the moments of inflation and unemployment growth are affected by changes in the inflation aversion parameter (λ) when there is a common stochastic trend in the employment targets (Panel A) and a stochastic trend in technology (Panel B). The reason for selecting this particular transformation of x_t, i.e. $\beta_u = 0$, is that we wish to keep the long run relation unaffected by changes in λ. Although the effects on the variance of unemployment growth are more complex in this setting, the picture is broadly the same as in Table 1. At low (high) values of λ (relative to ω/α), an increase in λ tends to lower (raise) the variance of unemployment growth while the mean and the variance of inflation decrease.

It should be noted that the inflation and unemployment equations above have been derived from a model where agents do *not* consider the possibility that the parameters may change. When we use our version of the Rogoff model to interpret the time series evidence below, we thus implicitly assume that the regime shifts we find are associated with small changes in these parameters.

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[8] Generally, the equilibrium level of employment is given by

$$n_t = n_{t-1}^u + \frac{1}{1+\alpha^2\lambda}\left(n_t^* - E_{t-1}n_t^*\right) + \frac{\alpha\lambda}{1+\alpha^2\lambda}\varepsilon_{z,t}.$$

[9] If we define a relation between unemployment and inflation as

$$u_t = u_t^n + \phi\left(\pi_t - E_{t-1}\pi_t\right),$$

with the natural rate of unemployment being $u_t^n \equiv n^s - n^u + \omega\alpha(n^d - n^u)$, then equations (9) and (11) imply that $\phi = \alpha\lambda - \omega$. Accordingly, if $\lambda < \omega/\alpha$, then the slope of this curve is negative. Reversely, if the above relation is negatively sloped, then a small increase in the central bank's inflation aversion will lower the variance of unemployment.

3. Unemployment and Inflation Regimes in the U. S.

In this section we shall discuss results for U. S. monthly data on unemployment and inflation. In relation to the general specification in (2) we shall focus on four issues. First, for a VAR model with s_t constant, is there any evidence of cointegration between inflation and unemployment? Second, does the constant (or single) regime model appear to be well specified? To address this question we shall perform some common misspecification tests. Third, we shall consider estimation of a cointegration relation under the assumption that the VAR model is subject to switching regimes. Finally, we will check which changes in the parameters from the theoretical model in Section 2.2 that are feasible explanations for the differences between the regimes.

3.1. *Single-Regime VAR Models*

The U. S. time series for the sample period 1959:1–1998:12 are portrayed in Figure 1. The inflation series is computed from the CPI (base year is 1967) for all urban consumers (U. S. city average, not seasonally adjusted) and is taken from the U. S. Department of Labor, Bureau of Labor Statistics. The series is in natural logarithms and measured as the monthly change in annual percent. The unemployment series is also taken from the Bureau of Labor Statistics and is measured as (100 times) the natural logarithm of the civilian labor force relative to the civilian employment (number of people). Both these labor market series are seasonally adjusted and are based on workers that are 16 years or older.

Figure 2 presents scatter plots of the data. The top panel contains the monthly inflation figures measured as a yearly inflation rate ($\pi_t = 12[p_t - p_{t-1}]$), while the bottom panel depicts yearly inflation ($\pi_t^{(12)} = p_t - p_{t-12}$). As expected, the monthly variation in inflation seems to be greater than the yearly. Moreover, both inflation measures as well as unemployment seem to have positively skewed distributions, where in particular large values for unemployment tend to coincide with small values for inflation. For other values, however, it is difficult to see any relation between the variables.

The results from testing for cointegration, i.e. a long run relation between these variables, are displayed in Table 3. The statistical model is a standard, single-regime VAR(k) with Gaussian errors, centered seasonal dummies, and the constant is restricted to the cointegration space; see e.g. Johansen (1995). The restrictions on the constant ensure that if there are unit roots in x_t, then the time series will not have a linear trend. According to the asymptotic distribution of the so called trace statistic (LR_{tr} in Panel A), there is evidence of one, but not two unit roots.[10]

[10] These results are not qualitatively affected by the exclusion of seasonal dummies in the VAR. The p-values have been computed using the software developed in connection with MacKinnon, Haug, and Michelis (1999) and can be downloaded from `http://qed.econ.queensu.ca/pub/faculty/mackinnon/johtest/`.

In Panel B we report tests of the hypothesis that either unemployment or inflation is stationary, conditional on a single unit root. For lag orders between 6 and 12, all hypotheses are rejected at the 5 percent level and only in the case of inflation at shorter lags is there an indication that the series may be stationary at the 1 percent level. The point estimates of the cointegration vector when we normalize the relation on inflation are presented in Panel C of Table 3. For lag orders between 6 and 12 the estimated coefficient on unemployment, $\hat{\beta}_u$, is positive and greater than unity. Hence, the cointegration analyses from the linear VAR models suggest that there is a positive long run relation between inflation and unemployment for the U. S. data.

This finding of a cointegration relation between (or, equivalently, a common stochastic trend in) inflation and unemployment should not be surprising. However, the estimate of the normalized cointegration relation yields a much larger value of β_u than reported by Ireland (1999). Moreover, in view of the theoretical model in Section 2.3 with a common trend in the employment targets, an estimate of β_u which is greater than 0.5 is unreasonable if we think that γ is close to unity.[11]

Furthermore, when we turn to the specification analysis in Table 4, we find that all these models, to various degrees, appear to suffer from serial correlation and/or conditional heteroskedasticity in the residuals. In Panel A we report two serial correlation tests, a system based Ljung-Box test and a system based *LM* test, and in Panel B, equation based ARCH tests; the column "# Unit Roots" refers to the number of unit roots that have been imposed on the system, e.g. zero unit roots is an unconstrained VAR(k) model for x_t.

The *LM* tests indicate that the VAR residuals are serially correlated for all lag orders at the 5 percent level, while the Ljung-Box tests suggest that a lag order of 8 may be sufficient to capture serial correlation in x_t. Moreover, the test results are only weakly influenced by a unit root restriction.

From Panel B we find evidence of kth order ARCH in both equations at the shorter lags and in the case of inflation also for the VAR(12) models. Hence, the standard, single-regime VAR model does not seem to be consistent with the U. S. data.

3.2. *Two-State Markov Switching VAR Models*

In this section we shall examine a VAR model with 2 regimes where the regime process, for simplicity, is assumed to follow an unobserved ergodic Markov chain. Visually inspecting the unemployment series suggests that the "jumps" may either be due to large shocks to a stochastic trend or to regime shifts (or both); see Figure 1:III. The finding of a unit root

[11] For example, if we let $\alpha = 1/3$, $\lambda = 1$, then $\gamma = .8$ implies that $\beta_u = 9/(15 + 5\omega)$, whereas $\gamma = 1.2$ implies that $\beta_u = -9/(15 + 5\omega)$. These ratios are closer to zero for larger values of λ (and α). Hence, as long as the labor supply function has a positive slope, the model predicts a value of β_u in the range $[-.5, .5]$ for "realistic" values of the theoretical parameters.

in the single regime VAR models for x_t may thus be spurious. On the other hand, if there are unit roots in x_t, there are several ways one can account for such a feature in an MS-VAR model.

Karlsen (1990) presents a sufficient condition for stationarity for a q-state MS-VAR(k); see also Holst, Lindgren, Holst, and Thuvesholmen (1994). With $q = 2$, let $e_1 \geq \ldots \geq e_{8k^2} \geq 0$ be the ordered eigenvalues (measured as e.g. the modulus) of the matrix

$$A = \begin{bmatrix} (A_1 \otimes A_1)p_{11} & (A_1 \otimes A_1)p_{21} \\ (A_2 \otimes A_2)p_{12} & (A_2 \otimes A_2)p_{22} \end{bmatrix}, \tag{12}$$

where A_{s_t} is the $2k \times 2k$ matrix obtained from a VAR(1) stacking of equation (2). Karlsen's condition for stationarity states that x_t is second order stationary if $e_1 < 1$. Similarly, if $e_1 = 1$ and $e_2 < 1$, then x_t has exactly one unit root.

A straightforward approach to imposing a unit root on the system in equation (2) is to first express it in an "error correction" form:

$$\Delta x_t = \delta_{s_t} + \sum_{i=1}^{k-1} \Gamma_{i,s_t} \Delta x_{t-i} + \Pi_{s_t} x_{t-1} + \varepsilon_t, \tag{13}$$

where $\Gamma_{i,s_t} = -\sum_{j=i+1}^{k} A_{j,s_t}$, $\Pi_{s_t} = \sum_{j=1}^{k} A_{j,s_t} - I_2 = \alpha_{s_t}\beta'$, with β being a 2×1 vector with rank 1.[12] Second, this system can be stacked in VAR(1) form, with autoregressive matrix Γ_{s_t}, and a new A matrix can be defined as in (12), but with A_{s_t} replaced with Γ_{s_t}. If $e_1 < 1$ for the new A matrix, then Δx_t and $\beta' x_t$ are stationary processes.

Alternatively, the MS-VAR model for x_t in (2) can be rewritten as an MS-VAR model for $y_t = C(\beta' x_t, S\Delta x_t)$, where the 2×2 matrices $B = (\beta', S)$ and C have rank 2 (for instance, $S = \beta'_\perp$ and $C = B^{-1}$), i.e.

$$y_t = \psi_{s_t} + \sum_{j=1}^{k} B_{j,s_t} y_{t-j} + \varphi_t, \tag{14}$$

where $\psi_{s_t} = CB\delta_{s_t}$, $\varphi_t = CB\varepsilon_t$, and B_{j,s_t} is a function of $(C, B, \Gamma_{j,s_t}, \Gamma_{j-1,s_t})$ for $j \geq 2$, while B_{1,s_t} depends on $(C, B, \Gamma_{1,s_t}, \alpha_{s_t})$. Stacking this system in VAR(1) form, with autoregressive matrix B_{s_t}, then y_t is stationary if $e_1 < 1$ for an A matrix based on B_{s_t} rather than A_{s_t}.[13]

For the U. S. data we find that the largest eigenvalue for an MS-VAR(3) model for x_t is about .962 and for an MS-VAR(2) model .974, thus suggesting that x_t may indeed have a unit root.

Maximum likelihood estimation of the parameters in (13) can be achieved via the EM algorithm (see e.g. Hamilton, 1990, 1994). One difficulty, relative to a model that is linear

[12] A special case of the error correction model in (13) is discussed by Krolzig (1996).

[13] The derivation of equation (14) is presented in the Appendix.

conditional on the regime (such as (2)), is the nonlinear relation involving α_{s_t} and β. In this paper we use a grid search procedure, where the grid is defined over the entries in β that can be uniquely determined. This means that estimation of (13) and (14) involves solving the same problem since both systems are linear conditional on β and on the regime.[14] We shall therefore only examine the representation in (14).

Specifically, we let $S = (0 \quad 1)$ so that $S\Delta x_t = \Delta u_t$ and for β we let the coefficient on inflation be equal to unity and vary β_u.[15] For each value of β_u in the grid, the free parameters defined by $(p_{ii}, \psi_i, B_{j,i}, \Sigma_i : i = 1,2; j = 1,\dots,k)$, where $\Sigma_{s_t} = CB\Omega_{s_t}B'C'$, are estimated via the EM algorithm and the corresponding value of the log-likelihood function is computed. The value of β_u which achieves the largest log-likelihood value is then selected as the estimate of β_u.

The grid search results from estimating a 2-state MS-VAR(2) model of y_t are summarized in Figure 3. In addition to the value of the log-likelihood function we have also plotted the largest eigenvalue for (14); the log-likelihood values have therefore been scaled in Figure 3.[16] This procedure gives us an estimate of β_u equal to .039, while the value of $\ln L$ is equal to -922.63.[17]

The estimate of the cointegration relation conditional on two regimes thus produces a much smaller and, in view of equation (8), more "realistic" value of β_u than what comes out of the single regime models. The estimate is also smaller than that obtained by Ireland (1999). This result may be interpreted in two ways:

(i) There is a common trend in inflation and unemployment due to a common trend in the employment targets. In this case, the small value for β_u is due to either a high value of λ, the central bank's inflation aversion,[18] or to γ being close to unity. For both possibilities, λ is constant across regimes.

(ii) Inflation is stationary and the stochastic trend in unemployment is due to either a common trend in the employment targets with $\gamma = 1$, or to a

[14] To make sure that the density function for φ_t is invariant with respect to B and C, these matrices should be selected such that $\det(CB) = \pm 1$. By selecting $C = B^{-1}$ this is always guaranteed. Alternatively, if B is upper triangular with unit diagonal elements, then we can always let C be equal to the identity matrix.

[15] This means that we can let $C = I_2$ and thus that $\det(CB) = 1$; see footnote 14.

[16] The scaling function is simply:

$$s\Big(\ln L\big(\beta_u\big)\Big) = 1 + \Big(\ln L\big(\beta_u\big) - \max_{\beta_u \in [-2,3]} \ln L\big(\beta_u\big)\Big)/20,$$

where the grid is specified over the interval $[-2, 3]$.

[17] For the MS-VAR model which does not impose the unit root, i.e. the system in (2) with $k = 3$ and $a_{i2,3,s_t} = 0$ for $i = 1,2$, the value of $\ln L$ is -914.66. Relative to the model in (14), this MS-VAR has 3 additional free parameters.

[18] In principle, it may also be due to high values of α and/or ω, but this seems less likely.

stochastic trend in technology. It is possible that changes in regime are due to changes in λ.[19]

We favor the second explanation, and during the remainder of the anaylysis of the U. S. data we will present results that support this idea.

In Table 5 we present specification tests and some system properties for 3 MS-VAR models. System 1 is defined from (14) with $y_t = (\pi_t - .039u_t, \Delta u_t)$ and $k = 2$, System 2 uses $y_t = (\pi_t, \Delta u_t)$, i.e. assumes that inflation is stationary, whereas System 3 is given by (2) with $k = 3$. In terms of the equation-by-equation tests in Panel A[20] the three MS-VAR systems behave satisfactorily. The system tests give a similar picture thus suggesting that an MS-VAR model with 2 states and a low lag order is consistent with the data.

In Panel C we report some system properties of the three MS-VAR models. Systems 1 and 2 generally display the same behavior, suggesting that conditional on a unit root inflation is stationary, whereas System 3 differs primarily in terms of its high maximum eigenvalue ($\hat{e}_1$ close to unity). Comparing these system properties to those of the linear VAR models (see Panel C in Table 3) we find that the information criteria are smaller for the MS-VAR models. This is often due to higher log-likelihood values as well as a lower dimension of the parameter vector. Given the better performance of the specification tests, these results support the view that for the U. S. data an MS-VAR model with 2-states and a low lag order is to be preferred over a single regime model with a higher lag order.

3.3. *Regime Properties of Inflation and Unemployment in the U. S.*

In this section we will first consider the robustness of the estimated regimes over small changes in the preselected parameters. Second, the estimated first and second moments conditional on the regime are presented, and, finally, we compare these to the effects of small changes in the parameters of the economic model.

The estimated smooth probabilities, i.e. $\Pr[s_t = 1|x_T, x_{T-1}, \ldots x_1; \hat{\theta}]$, are displayed for 4 models in Figure 4. In Graph I the model is given by (2) with 3 lags and zero restrictions on the 3rd lag for the parameters on unemployment; Graph II gives the estimated state 1 probabilities for a 2 lag version of (2) with zero restrictions on the 2nd lag of unemployment; Graph III contains the estimates for a 2 lag model of the type in equation (14) with inflation stationary while unemployment has a unit root (i.e. this model is the same as the model in Graph I but with a unit root restriction and with $\beta_u = 0$); and finally Graph IV presents the

[19] Ireland (1999) reports that inflation is indeed a borderline case and may very well be stationary. His theoretical model does not, however, allow for the possibility of different stochastic trends in inflation and unemployment. The reason is that the sources behind the trend in unemployment are not modeled.

[20] See Hamilton (1996) for details on the setup of the three hypotheses for the F-versions of the conditional scores test due to Newey (1985), Tauchen (1985), and White (1987). Note that $\varpi_j = \Pr[s_t = j]$, the ergodic probability of being in Regime j.

estimates for a 2 lag version of (14) with inflation and unemployment cointegrating with the coefficient on unemployment equal to 1.84 (the Johansen ML estimate from the VAR(12) model).

The four models give very similar estimates of the smooth probabilities. The major difference is the period between late 1975 and the end of 1979. Here the model estimates in Graphs I and III suggest that the regime process remains in Regime 1, whereas the estimates in Graphs II and IV prefer Regime 2. From a statistical point of view, the models that yield the plots in Graphs I and III are to be preferred.[21] Comparing these estimates to the case when $\beta_u = .039$ (the grid estimate), we find that the smooth probabilities are virtually the same as those for the $\beta_u = 0$ case. The maximum posterior estimates of the regime process are taken from the $\beta_u = 0$ model, and these regime estimates are displayed in Figure 1, where the shaded areas represent Regime 2.[22]

In Table 6 we present the estimated unconditional (Panel A) and conditional (Panel B) moments of y_t systems under the grid estimate of β_u and under the assumption that inflation is stationary while unemployment has a unit root. The conditional moments refer to conditioning on the current state only, e.g. the conditional mean is $E[y_t|s_t]$; analytical formulas and the estimation of such moments is examined by Warne (1999b). From Panel B it can be seen that inflation (or the cointegrating relationship) tends to be higher on average in Regime 1 and also more volatile than during Regime 2. Similarly, unemployment is typically rising in Regime 1 and falling during Regime 2.[23] Hence, Regime 1 (Regime 2) can be characterized as a high (low) inflation, rising (falling) unemployment regime with large (small) variances.

In Table 2 we give the effects on the mean and the variance of inflation and unemployment growth from changes in the wage elasticity of labor supply (ω) and the central bank's weight on inflation (λ). The theoretical predictions from changes in these parameters are quite similar under both types of nonstationarity (cf. Panels A and B in Table 2). In particular, if the central bank's weight on inflation (λ) is higher in Regime 2 than in Regime 1, shifting from Regime 1 to Regime 2 results in a lower mean and variance of inflation as we have found for the U. S. data. According to both theoretical models, average unemployment growth will, however, not be affected.

The evidence in Table 6 is thus consistent with the predictions in Table 2 about the mean and the variance of inflation and the mean of unemployment for changes in λ (but not for

[21] There are signs of model misspecification for the 2 lag model of x_t, while the model with $\beta_u = 0$ has a much higher log-likelihood value than the model with $\beta_u = 1.84$.

[22] The maximum posterior estimate for s_t is defined by

$$\hat{s}_t = \arg\max_{i=\{1,2\}} \Pr\left[s_t = i \,\middle|\, x_T, x_{T-1}, \dots, x_1; \hat{\theta}\right], \qquad t = k+1, \dots, T.$$

[23] The standard errors are computed using the delta method with numerical partial derivatives.

ω). Moreover, according to Table 2, an increase in λ may raise or lower the variance of unemployment growth depending on how large λ is in relation to ω and α (capital's share of value added). The estimated MS-VAR model for y_t with inflation stationary results in a lower variance for unemployment in the low-inflation regime. According to the theoretical model, this is either due to λ being small or ω being large.

4. Unemployment and Inflation Regimes in Sweden and the U. K.

In this section we shall compare the results for the U. S. to Sweden and the U. K. Our primary concern is if the finding that U. S. unemployment volatility is lower in the low than in the high inflation regime also holds for these two European economies. The econometric analysis is performed in the same manner as for the U. S., so we shall only summarize our findings here.

The Swedish data covers the sample period 1970:2–1998:12 on the log of monthly CPI inflation (base year is 1949) and unemployment, measured as the log of total number of employed plus unemployed relative to the number of employed (Statistics, Sweden). For the U. K. we look at monthly data from 1950:4–1998:12 on the log of monthly RPI (retail price index) inflation (base year is 1947; Bank of England), while the unemployment series is calculated from data on the U. K. work force and the number of employed (Office for National Statistics) and additional data on the number of unemployed from the OECD Main Economic Indicators and the U. K. Administrative unemployment level (see Denman and McDonald, 1996). Details are given in the Data Appendix.

The data is graphed in scatter plots in Figure 5. In the case of Sweden, there is a large cluster of observations around the 2.5 percent unemployment rate and an inflation rate of 8 percent, and a smaller cluster with a lower inflation rate and a higher and more volatile unemployment rate. If these two clusters are viewed as two separate regimes, then this graph suggests that unemployment volatility indeed is higher in the low inflation regime than in the high inflation regime. Moreover, inflation is more volatile in the latter regime. Alternatively, the graph may reflect a negative long run relation between unemployment and inflation.

For the U. K., on the other hand, the relation between unemployment and inflation looks more like the U. S. case (cf. Figure 2, Graph II). The main differences are the relatively large number of observations of high unemployment for the U. K. In a linear setting, the U. K. data may reflect that there is not any long run relation between unemployment and inflation.

4.1. *Single-Regime VAR Models*

The evidence from our cointegration analyses for Sweden and the U. K. is summarized in Table 7. The tests for the number of unit roots in the linear VAR models (with restrictions on the constant term) in Panel A generally support the hypothesis that there is one unit root, i.e., that there is a stationary linear combination between inflation and unemployment. Only when we study a 12 lag model using the Swedish data do we find stronger support for 2 unit roots.

Turning to the stationarity tests in Panel B (where the models have been conditioned on a single unit root), the results suggest that unemployment is not stationary. In the case of Sweden, we can also reject the hypothesis that inflation is stationary, while for the U. K. the hypothesis cannot be rejected at the 5 percent level.

The estimated coefficient on unemployment for the normalized cointegration vector is reported in Panel C. For Sweden, the value is very stable (roughly -1.25), thus suggesting that there is a negative long run relation between inflation and unemployment. In contrast, the U. K. estimates are positive, like for the U. S. (cf. Table 3), but close to zero. Both these results are consistent with the evidence from the stationarity tests in Panel B. Moreover, in view of the plots in Figure 5 the results are hardly surprising when we analyse the data with linear models.

However, these linear models do not seem to adequately describe the behavior of the data. In the case of Sweden, there is some evidence of serial correlation when we test for autocorrelation at the 12:th lag and strong evidence of ARCH, particularly in the unemployment equation. For the U. K., there is overwhelming evidence of serial correlation in the estimated VAR residuals and also very strong indications of ARCH in the unemployment equation residuals.[24] Hence, the standard, single-regime VAR model does not seem to provide a sufficiently good representation of the Swedish or of the U. K. data.

4.2. *Two-State Markov Switching VAR Models*

Some of the properties from estimating MS-VAR(k) models for Sweden and the U. K. are presented in Table 8. System 3 corresponds to the levels models in (2), where we have selected 2 lags for Sweden and 3 for the U. K. In the Swedish case we find that the value of the log-likelihood function is roughly equal to that of a single regime VAR with 8 to 10 lags. Comparing the estimates of the three information criteria for the 2-state model to those for the single regime models, we find that such criteria strongly prefer the 2-state model. For the U. K. we even find that the 2-state MS-VAR(3) model yields a much higher value for the log-likelihood than the VAR(12) model does.

As in the U. S. case, we have also computed specification tests for the MS-VAR models for Sweden and the U. K.[25] Here, we also find that the regime switching models seem to do a

[24] The specification test results are available from the authors on request. Moreover, at this time they may also be available for download from the Riksbank's web site (`http://www.riksbank.com/`) and from Anders Warne's personal home page (`http://www.farfetched.nu/anders/`).

[25] See footnote 24.

better job describing the behavior of the data than the single regime models do. There are no longer any signs of serial correlation or additional conditional heteroskedasticity and the Markov tests cannot be rejected. Moreover, the MS-VAR models are able to accomplish this without having to increase the dimensionality of the systems.

Still, there are strong signs of unit roots in the regime switching models. In fact, for the Swedish data the largest eigenvalue of the estimated A matrix, denoted by $\hat{e}_1$, is greater than unity (System 3 in Table 8, Panel A). We therefore impose a unit root in each case, estimate the cointegration relation via the grid search procedure, and check what the largest eigenvalue is for the selected system. For Sweden we find that the estimated coefficient on unemployment is roughly minus 1 while the U. K. coefficient is very close to zero. The resulting log-likelihood value, information criteria, and largest eigenvalue are found in Table 8 under System 1.

Typically, the likelihood loss is quiet modest and the estimated largest eigenvalues well below unity. The scaled log-likelihood function over a selected range for β_u and the corresponding largest eigenvalues are portrayed in Figure 6 for Sweden (Graph I) and the U. K. (Graph II). Hence, it seems appropriate to consider a regime switching model for inflation and the first difference of unemployment for the U. K. data (System 2 in Table 8). For comparison reasons we shall also examine such a model for Sweden; it may be noted that the log-likelihood loss is quite large, but the model (System 2) does not display any signs of nonstationarity ($\hat{e}_1 = .26$) and it passes all the specification tests.[26]

The estimated moments are found in Table 9. The means and the variances of inflation are roughly the same in Sweden and the U. K. (and higher than the U. S. estimates). Unemployment growth is roughly zero for both economies, while the variance of this time series is roughly twice as high for Sweden than for the U. K. (and the U. S.). Turning to the regimes we find that inflation is higher and more volatile in Regime 1 than in Regime 2 for both countries. In contrast with the U. S., it can also be seen from the Table that the variance of unemployment growth is higher in the low inflation regime than in the high inflation regime.

The latter result is consistent with λ, the central bank's weight on inflation in its objective function, being higher in the low inflation regime (Regime 2) than in the high inflation regime (Regime 1); cf. Tables 1 and 2. In comparison with the U. S. result, this may either be interpreted as λ being higher in Sweden and the U. K., or that ω, the wage elasticity of labor supply, is lower in the European countries than in the U. S.

[26] We find that the ARCH test for the $(\pi_t + .957u_t)$ equation is rejected at the 5 percent level (System 1), but not for the π_t equation (System 2). Hence, there is also some statistical evidence favoring the MS-VAR model (14) with $y_t = (\pi_t, \Delta u_t)$ over the model with $y_t = (\pi_t + .957u_t, \Delta u_t)$ for Sweden.

5. Relations to Some Earlier Literature

Our analysis of unemployment and inflation is in many ways similar to the analyses applied in Ireland's (1999) and King and Watson's (1994) studies of U. S. data, but there are also important differences.

Ireland (1999) proposes that inflation and unemployment are both nonstationary, although inflation is a borderline case, and cointegrated. Ireland suggests that there is a stochastic trend in the natural level of unemployment that is translated into a stochastic trend in inflation. The reasons for this are that the Fed's unemployment target differs from the natural rate and that monetary policy cannot commit to the inflation target. Ireland thus claims that U. S. inflation rose (fell) before (after) 1980 because of changes in the natural rate of unemployment, and that monetary policy has been stable over time.

King and Watson (1994) also suggest that U. S. inflation and unemployment are nonstationary, but they do not find them to be cointegrated. They stress that the links between inflation and unemployment are unstable over time. In particular, they emphasize that a distinguishing feature of the 1970–92 period (the last part of their sample) is persistence in the effects of shocks. Ireland (1999) also stresses that the persistence in the inflation-unemployment relation cannot be captured by his theoretical model. King and Watson's results are largely consistent with the more recent study by Stock and Watson (1999), who however also note that inflation may very well be stationary.

Our results show that the cointegration relation between inflation and unemployment is much weaker when we allow for changes in regime. Conditional on the regimes, we find that inflation and unemployment may be cointegrated, but in that case the "long run Phillips curve" is almost horizontal and we may as well treat inflation as stationary. Our results thus suggest an interpretation of the U. S. time series which is different from Ireland's. We do find a stochastic trend in unemployment, but also find it likely that it does not affect inflation. This will, e.g., be the case if the Fed's employment target moves one for one with changes in the natural rate of employment,[27] or if technology (the Solow residual) is the source of nonstationarity. This still leaves open the possibility that high and low inflation regimes reflect different preferences regarding inflation and employment stabilization (λ). The period 1973–83 may thus have been a high inflation regime because the Fed put relatively more weight on employment during this period. Analogously, 1991–98 may have been a low inflation regime because the weight on employment has been relatively low.

If a central bank puts a larger weight on employment stabilization, many theoretical models predict that inflation persistence will increase (e.g. Svensson, 1999). King and Watson's finding that the effects of shocks were very persistent during the period 1970–92 is thus

[27] In our theoretical model, this means that $\gamma = 1$ and that the Fed's employment target deviates from the wage setters' employment target by a constant.

consistent with our interpretation of why the U. S. economy was in the high inflation regime during most of that period.

Sims (1999) estimates a Markov switching model of a relation between a nominal interest rate and the level of commodity prices using U. S. data. This relationship and the regime switches are interpreted as reflecting monetary policy. Sims finds that the response to inflation is relatively weak in a high interest rate state, and that the variance of the interest rate varies across regimes. These findings, and some of Sims's other arguments, are consistent with ours.

Gruen et al. (1999) apply a Markov switching model to Australian data on inflation and unemployment. They distinguish between a high unemployment state (above NAIRU) and a low unemployment state (below NAIRU) which through their model of the Phillips curve are associated with relatively low and high inflation, respectively. In contrast to our analysis, their model does not allow for regime shifts in monetary policy.

Haldane and Quah (1999) argue that the (long run) Phillips curve has been practically horizontal in the U. K. since 1980, which they interpret as a result of purposeful monetary policy by policy makers who believe in the existence of a natural rate of unemployment. Their theoretical framework is essentially the same as ours and Ireland's. In contrast to the arguments made in our paper (and by Sims, 1999), however, Haldane and Quah (and Ireland) view monetary policy as quite stable over long periods of time. We find a horizontal (long run) Phillips curve to be a good and stable approximation of the long run properties between inflation and unemployment in both the U. S. and the U. K. because inflation rates can be treated as stationary while unemployment rates cannot. But we also show that this (which occurs in the theoretical model when either $\gamma = 1$, or when the only source of nonstationarity is due to a stochastic trend in technology) does not rule out changes in monetary policy (changes in λ). In the short run, within each policy regime, the Phillips relation may appear to be negatively or positively sloped, or horizontal or vertical. In our interpretation, this is only a small sample phenomenon. As an illustration we show scatter plots of inflation and unemployment for the regimes we have estimated in the U. S. data; see Figure 7.

As noted by Haldane and Quah (and many others), there is much confusion in the literature about Phillips curves because economists put this label on quite different economic relations. In this paper we have tried to avoid this terminology as much as possible, or at least to be clear about what we mean. The unconditional correlation between inflation and unemployment is a meaningful concept if inflation and unemployment are both stationary. If they are not, a "long run Phillips curve" may be estimated as a cointegration relation, where the coefficients on inflation and unemployment determine if the slope is negative, positive, horizontal, or vertical. We suggest that it is reasonable to treat inflation as stationary while unemployment may very well be nonstationary. This may thus be described as a horizontal long run Phillips curve. But, as noted by Haldane and Quah, even in this case there may be a negatively sloped or vertical "Phillips curve" between inflation and unemployment, conditional on expected inflation and the natural rate of unemployment. The version of Rogoff's (1985) model that we use tell us how the slope of that relation depends on labor demand and supply and what factors determine the natural rate of unemployment.[28] We believe that differences between countries regarding such labor market conditions may explain some of our findings, but we dot identify any structural parameters from our empirical model.

6. Summary and Conclusions

In this paper we address the issue if high-inflation regimes are characterized by more or less unemployment volatility. Previous studies of this question (such as Alesina and Summers, 1993, and Jonsson, 1995) have analyzed cross-country data and have failed to detect a relation between the mean of inflation and the variance of unemployment. If there have been, e.g., monetary policy changes within countries such data are likely to overlook important time series information.

One feature that needs to be defined in a time series framework is what we mean by "high" and "low" inflation. Assuming that the data are stationary, the unconditional means and variances and inflation and unemployment will not be useful. Rather, we suggest that the means and variances conditional on the current regime should be examined. It thus remains to either "observe" or specify a model for the regimes. Here, we use a simple approach by assuming that the regimes cannot be observed, that they are exogenously determined with respect to inflation and unemployment, and follow a first order Markov process. For such regime processes we know how to compute and evaluate the conditional means and variances; see Timmermann (2000) and Warne (1999b).

To interpret such estimates we have presented an extended version of Rogoff's (1985) model of monetary policy. First of all, our version is consistent with equilibrium unemployment. Second, it has the realistic feature of allowing both unemployment and inflation to be persistent. For example, under certain assumptions they can both be nonstationary and cointegrated.

Typically, the model predicts that if the central bank puts a low (high) weight on inflation (relative to the labor supply elasticity) in its objective function, then a small increase in this parameter tends to lower (raise) the variance of unemployment (growth). Such changes in the inflation weight parameter also tends to lower the mean and the variance of inflation, while the mean of unemployment (growth) is not affected. The effects on the means and

[28] See footnote 9 and the Appendix.

variances of inflation and unemployment (growth) from changes in the other parameters of the Rogoff model are typically quite different.[29]

To examine the relation between inflation and unemployment volatility empirically we have analyzed monthly data for the U. S., the U. K., and Sweden. In all three countries the variance of inflation is higher in the high-inflation than in the low-inflation regime. For Sweden and the U. K., the variance of unemployment is higher in the low-inflation regime, while it is lower for the U. S. These results are thus consistent with the predictions of the economic model provided that either the weight on inflation in the central bank's objective function is higher in the European countries, or the labor supply elasticity is higher for the U. S.

These particular economic interpretations depend on the assumption that inflation is stationary (unemployment, however, may be stationary or nonstationary). For the U. S. and the U. K., there is strong evidence supporting this hypothesis when we allow for changes in regime, i.e., the "long run Phillips curve" is horizontal. The empirical evidence for Sweden is much weaker and, in fact, the MS-VAR model tends to favor a negative long run relation between inflation and unemployment (with the coefficient -1 on unemployment). Such a relation is harder to interpret in our framework.

The regime changes we have in mind are smaller and occur more often than the large deterministic changes in regime usually considered in analyses of macroeconomic time series. We do believe that small changes in monetary policy, e.g., preferences for inflation stability relative to employment stability, are important. Nevertheless, our analyses may have abstracted from other important shocks to inflation and unemployment, e.g., fiscal policy and commodity prices. Such factors may be especially important in a small open economy with a large public sector. This can be a reason why the results for Sweden are somewhat harder to interpret than the results for the U. S. and the U. K.

[29] The only parameter that can, at least in theory, affect these moments in a similar fashion is capital's share of value added.

Appendix

Derivation of the Economic Model

Below we present an extension of Rogoff's (1985) model. The presentation follows Rogoff closely, and where notation is obvious we leave out detailed explanations.

Production is determined by a Cobb-Douglas function

$$y_t = \alpha\kappa + (1-\alpha)n_t + z_t, \tag{A.1}$$

where κ is the fixed capital stock and technology, z_t, follows the process

$$z_t = \rho z_{t-1} + \varepsilon_{z,t}, \tag{A.2}$$

with $\varepsilon_{z,t}$ being white noise with mean zero and variance σ_z^2. Profit maximization, taking p_t and w_t as given, yields the labor demand function

$$n_t^D = n^d - \frac{1}{\alpha}(w_t - p_t) + \frac{1}{\alpha}z_t, \tag{A.3}$$

where $n^d = \kappa + \ln(1-\alpha)/\alpha$. The (notional) labor supply function is assumed to be given by

$$n_t^S = n^s + \omega(w_t - p_t). \tag{A.4}$$

However, the wage is set at w_t^f in period $t-1$, and labor is supplied infinitely elastically at that wage in period t. Hence, employment in period t is given by

$$n_t = n^d - \frac{1}{\alpha}(w_t^f - p_t) + \frac{1}{\alpha}z_t. \tag{A.5}$$

The wage is set in order to minimize $E_{t-1}(n_t - n_{t-1}^u)^2$, where n_{t-1}^u is the wage setters' employment target in period $t-1$ for period t. The nominal wage for period t is therefore

$$w_t^f = E_{t-1}p_t + \alpha(n^d - n_{t-1}^u) + \rho z_{t-1}. \tag{A.6}$$

The central bank's objective function is given by

$$\Lambda_t = (n_t - n_t^*)^2 + \lambda(\pi_t - \pi^*)^2, \tag{A.7}$$

where n_t^* is the central bank's employment target for period t. Minimizing Λ_t with respect to p_t, using (A.5), gives

$$p_t = \frac{\dfrac{w_t^f}{\alpha^2} + \dfrac{1}{\alpha}\left(n_t^* - n^d - \dfrac{z_t}{\alpha}\right) + \lambda(p_{t-1} + \pi^*)}{\lambda + \dfrac{1}{\alpha^2}}. \tag{A.8}$$

Rational expectations, (A.6) and (A.8) imply

$$E_{t-1}p_t = p_{t-1} + \pi^* + \frac{1}{\alpha\lambda}(E_{t-1}n_t^* - n_{t-1}^u). \tag{A.9}$$

Using (A.6) and (A.9) in (A.8) and defining $\pi_t = p_t - p_{t-1}$ then yields

$$\pi_t = \pi^* - \frac{1}{1+\alpha^2\lambda}\varepsilon_{z,t} + \frac{1}{\alpha\lambda(\lambda+\alpha^{-2})}\left(\lambda n_t^* + \frac{1}{\alpha^2}E_{t-1}n_t^*\right) - \frac{1}{\alpha\lambda}n_{t-1}^u. \tag{A.10}$$

Defining unemployment as $u_t = n_t^S - n_t$ (noting that $w_t = w_t^f$) and using (A.6) we get

$$u_t = n^s - n_{t-1}^u + \omega\alpha(n^d - n_{t-1}^u) + \left[\omega + \frac{1}{\alpha}\right](E_{t-1}p_t - p_t) + \omega\rho z_{t-1} - \frac{1}{\alpha}\varepsilon_{z,t}. \tag{A.11}$$

Inserting (A.9) and (A.10) into (A.11) gives

$$u_t = n^s - n_{t-1}^u + \omega\alpha(n^d - n_{t-1}^u) - \frac{1+\alpha\omega}{1+\alpha^2\lambda}(n_t^* - E_{t-1}n_t^*) + \omega\rho z_{t-1} + \frac{\omega - \alpha\lambda}{1+\alpha^2\lambda}\varepsilon_{z,t}. \tag{A.12}$$

The components in unemployment are due to employment being determined by

$$n_t = n_{t-1}^u + \frac{1}{1+\alpha^2\lambda}(n_t^* - E_{t-1}n_t^*) + \frac{\alpha\lambda}{1+\alpha^2\lambda}\varepsilon_{z,t}, \tag{A.13}$$

while labor supply is

$$n_t^S = n^s + \omega\alpha(n^d - n_{t-1}^u) - \frac{\alpha\omega}{1+\alpha^2\lambda}(n_t^* - E_{t-1}n_t^*) + \omega\rho z_{t-1} + \frac{\omega}{1+\alpha^2\lambda}\varepsilon_{z,t}. \tag{A.14}$$

The assumptions $n_t^* = n^*$, $n_t^u = n^u$, and $\rho = 0$ yield the inflation and unemployment relations in equations (9) and (11).

A Stochastic Trend in the Employment Targets

Suppose that the employment targets evolve according to the process

$$n_t^u = n_{t-1}^u + \varepsilon_{u,t}, \tag{A.15}$$

$$n_t^* = n^* + \gamma n_{t-1}^u, \tag{A.16}$$

where $\varepsilon_{u,t}$ is white noise with mean zero and variance σ_u^2. In addition, suppose that $\rho = 0$. From (A.10) we find that

$$\pi_t = \pi^* + \frac{1}{\alpha\lambda}n^* + \frac{\gamma - 1}{\alpha\lambda}n_{t-1}^u - \frac{1}{1+\alpha^2\lambda}z_t,$$

while (A.12) provides us with

$$u_t = n^s + \omega\alpha n^d - (1+\alpha\omega)n_{t-1}^u + \frac{\omega-\alpha\lambda}{1+\alpha^2\lambda}z_t.$$

Hence, unemployment is nonstationary and driven by the stochastic trend in wage setters' employment target, while the linear combination $\pi_t - \beta_u u_t$ is stationary with

$$\beta_u = \frac{1-\gamma}{\alpha\lambda(1+\alpha\omega)}.$$

We thus find that inflation is stationary if (and only if) $\gamma = 1$. Moreover, the sign of the slope of the long run Phillips curve depends entirely on how big γ is.

Moreover, if we define a short run Phillips curve according to

$$u_t = u_t^n + \phi(\pi_t - E_{t-1}\pi_t), \tag{A.17}$$

with the natural rate of unemployment being given by

$$u_t^n \equiv n^s + \omega\alpha n^d - (1+\alpha\omega)n_{t-1}^u$$

then the slope of the short run Phillips curve is given by $\phi = \alpha\lambda - \omega$. The model thus exhibits a negatively sloped curve if $\lambda < \omega/\alpha$.

A Stochastic Trend in Technology

Suppose instead that $n_t^u = n^u$, $n_t^* = n^*$, while $\rho = 1$. From (A.10) we now have that

$$\pi_t = \pi^* + \frac{1}{\alpha\lambda}\Big(n^* - n^u\Big) - \frac{1}{1+\alpha^2\lambda}\varepsilon_{z,t},$$

while (A.12) gives us

$$u_t = n^s - n^u + \omega\alpha(n^d - n^u) + \omega z_{t-1} + \frac{\omega - \alpha\lambda}{1+\alpha^2\lambda}\varepsilon_{z,t}.$$

Inflation is thus stationary, while unemployment is nonstationary and driven by the stochastic trend in technology. Accordingly, $\beta_u = 0$.

In this case, there are two ways we can define the natural rate of unemployment for (A.17). With

$$u_t^n \equiv n^s - n^u + (n^d - n^u) + \omega z_{t-1},$$

we find that $\phi = \alpha\lambda - \omega$. If instead we let

$$u_t^n \equiv n^s - n^u + (n^d - n^u) + \omega z_t,$$

then $\phi = \alpha\lambda(1+\alpha\omega)$. Hence, the latter definition leads to a positively sloped short run Phillips curve, while the former definition is consistent with both a negative and a positive slope parameter.

Derivation of the MS-VAR Model for y_t

Let x_t be an n dimensional vector time series satisfying equation (13) with α_{s_t} and β being $n \times r$ matrices with rank $r < n$. Define the matrix polynomials $D(L) = I_n - DL$ and $\Delta(L) = I_n - D_\perp L$, where the $n \times n$ matrix D is given by

$$D = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}.$$

Hence, $D(L)\Delta(L) = \Delta(L)D(L) = \Delta I_n$. Premultiplying equation (13) by B and using the above we obtain

$$D(L)\Delta(L)Bx_t = B\delta_{s_t} + \sum_{i=1}^{k-1} B\Gamma_{i,s_t}B^{-1}D(L)\Delta(L)Bx_{t-i} + B\alpha_{s_t}\beta' x_{t-1} + B\varepsilon_t. \tag{B.1}$$

Defining $\nu_t = \Delta(L)Bx_t$ and $\eta_{s_t} = [B\alpha_{s_t} \quad 0]$, we can rewrite equation (B.1) as

$$\nu_t - D\nu_{t-1} = B\delta_{s_t} + \sum_{i=1}^{k-1} B\Gamma_{i,s_t}B^{-1}\nu_{t-i} - \sum_{i=2}^{k} B\Gamma_{i-1,s_t}B^{-1}D\nu_{t-i} + \eta_{s_t}\nu_{t-1} + B\varepsilon_t. \tag{B.2}$$

Next, by rearranging terms, defining $y_t = C\nu_t$, $\psi_{s_t} = CB\delta_{s_t}$, $\varphi_t = CB\varepsilon_t$, and premultiplying both side of (B.2) by C we get

$$\begin{aligned} y_t = \psi_{s_t} &+ C\Big[B\Gamma_{1,s_t}B^{-1} + D + \eta_{s_t}\Big]C^{-1}y_{t-1} + \sum_{i=2}^{k-1} C\Big[B\Gamma_{i,s_t}B^{-1} - B\Gamma_{i-1,s_t}B^{-1}D\Big]C^{-1}y_{t-i} \\ &+ C\Big[-B\Gamma_{k-1,s_t}B^{-1}D\Big]C^{-1}y_{t-k} + \varphi_t. \end{aligned} \tag{B.3}$$

From equation (B.3) the mapping between the coefficients in (13) and (14) follows.

Data Appendix

Prior to estimating the MS-VAR models on the U. S. data we have removed seasonal fluctuations in the CPI inflation series through monthly dummy variables.

For Sweden, the unemployment series contains strong seasonal cycles. Moreover, there seems to have been a change in the pattern between May and June 1992. We have therefore removed seasonals from this series by running separate regressions for the 1970:1–1992:5 and 1992:6–1998:12 periods. Seasonals in CPI inflation have then been treated in the same way as for the U. S. data.

We have relied on various sources in the construction of a U. K. unemployment series. The main problem is that there are not any monthly observations on employment and the labor force until 1992:4. The number of unemployed from 1948:1–1998:12 have been computed by adding the OECD main economic indicators data on the number of unemployed from 1995:12–1998:12 to the 1948:1–1995:11 data on the U. K. Administrative unemployment level data (see Denman and McDonald, 1996).

Next, the employment series (1992:4–1998:12) is measured by the total number of employed (Labour Market Structure: Office for National Statistics). For the 1950:4–1992:3 observations on the number of employed we have computed them from the difference between the number of people in the labor force and the number of unemployed.

The labor force series for 1992:4–1998:12 period has been calculated by adding the number of employed to the number of unemployed. The labor force data for the 1950:4–1992:3

period is determined, using quarterly observations, by multiplying the total working population (Office for National Statistics) by 1.144 for the 1950:4–1967:12 period and by multiplying the U. K. work force jobs series (Office of National Statistics) by 1.1 for the 1968:1–1992:3 period. The coefficient 1.1 is given by the average ratio between the sum of the number of employed and unemployed and the U. K. work force jobs during overlap period 1992:4–1992:6. The 1.144 coefficient has been set in a similar fashion by comparing the new labor force data to the total working population data during the overlap period 1968:1–1968:12.

The unemployment series has then been calculated as the log ratio of the number of people in the labor force to the number of employed (multiplied by 100). Seasonals in this series have been removed by running a regression on monthly dummy variables. Prior to estimating the MS-VAR models, seasonal fluctuations in RPI inflation have been removed in the same way as for the U. S. data.

TABLE 1: Effects on the mean and the variance of inflation and unemployment from changes in the theoretical parameters when the inflation bias is positive ($n^* > n^u$).

		Inflation		Unemployment	
Parameter	Interpretation	Mean	Variance	Mean	Variance
π^*	inflation target	+	0	0	0
n^*	central bank's employment target	+	0	0	0
n^u	wage setters' employment target	–	0	–	0
λ	central bank's weight on inflation	–	–	0	?
α	capital's share of value added	–	–	?	?
n^s	labor supply	0	0	+	0
κ	capital stock	0	0	+	0
ω	wage elasticity of labor supply	0	0	?	?
σ_z^2	variance of supply shock	0	+	0	+

NOTES: If n^d is greater (less) than n^u, then the mean of unemployment is increasing (decreasing) in ω (cf. equation (5)). Similarly, the mean of unemployment is increasing (decreasing) in α if $n^d + 1/(1-\alpha)$ is greater (less) than n^u. The variance of unemployment is equal to $V(u_t) = \left(\omega - \alpha\lambda\right)^2 \sigma_z^2 / \left(1 + \alpha^2\lambda\right)^2$. This variance is increasing (decreasing) in ω if λ is less (greater) than ω/α; it is increasing in α if $\lambda \in \left(\omega/\alpha, 2(\omega/\alpha) + 1/\alpha^2\right)$ and decreasing in α if $\lambda < \omega/\alpha$ or $\lambda > 2(\omega/\alpha) + 1/\alpha^2$; and it is increasing (decreasing) in λ if λ is greater (less) than ω/α.

TABLE 2: Effects on the mean and the variance of inflation and unemployment growth from changes in ω and λ.

(A) *Stochastic Trend in Employment Targets*

Moment	Wage Elasticity (ω)	Inflation Weight (λ)
$E(\pi_t)$	0	$\frac{-n^*}{\alpha\lambda^2}$
$V(\pi_t)$	0	$\frac{-2\alpha^2\sigma_z^2}{(1+\alpha^2\lambda)^3}$
$E(\Delta u_t)$	0	0
$V(\Delta u_t)$	$2\alpha(1+\alpha\omega)\sigma_u^2 + \frac{4(\omega-\alpha\lambda)\sigma_z^2}{(1+\alpha^2\lambda)^2}$	$\frac{4\alpha(1+\alpha\omega)(\alpha\lambda-\omega)\sigma_z^2}{(1+\alpha^2\lambda)^3}$

(B) *Stochastic Trend in Technology*

Moment	Wage Elasticity (ω)	Inflation Weight (λ)
$E(\pi_t)$	0	$\frac{-(n^*-n^u)}{\alpha\lambda^2}$
$V(\pi_t)$	0	$\frac{-2\alpha^2\sigma_z^2}{(1+\alpha^2\lambda)^3}$
$E(\Delta u_t)$	0	0
$V(\Delta u_t)$	$\frac{2[(1+\alpha^4\lambda^2)\omega-(1-\alpha^2\lambda)\alpha\lambda]\sigma_z^2}{(1+\alpha^2\lambda)^2}$	$\frac{2\alpha(1+\alpha\omega)[(2+\alpha\omega)\alpha\lambda-\omega]\sigma_z^2}{(1+\alpha^2\lambda)^3}$

NOTES: Panel A is based on the assumptions in equations (A.15) and (A.16) with $\gamma = 1$ and, for simplicity, $\rho = 0$. The former implies that $\beta_u = 0$ so that inflation is stationary. Now,

$$\pi_t = \pi^* + \frac{1}{\alpha\lambda}n^* - \frac{1}{1+\alpha^2\lambda}\varepsilon_{z,t},$$

while

$$\Delta u_t = -\left(1+\alpha\omega\right)\varepsilon_{u,t-1} + \frac{\omega-\alpha\lambda}{1+\alpha^2\lambda}\Delta\varepsilon_{z,t}.$$

The partial derivatives in Panel B are derived under the assumptions $n_t^u = n^u$, $n_t^* = n^*$, while $\rho = 1$. Here, $\beta_u = 0$ with

$$\pi_t = \pi^* + \frac{1}{\alpha\lambda}\left(n^* - n^u\right) - \frac{1}{1+\alpha^2\lambda}\varepsilon_{z,t},$$

and

$$\Delta u_t = \frac{\omega-\alpha\lambda}{1+\alpha^2\lambda}\varepsilon_{z,t} + \frac{\alpha\lambda\left(1+\alpha\omega\right)}{1+\alpha^2\lambda}\varepsilon_{z,t-1}.$$

TABLE 3: Cointegration analysis for bivariate VAR(k) models of inflation and unemployment for the U. S., 1959:1–1998:12

(A) *Cointegration Tests*

# lags	# Unit Roots	Eigenvalue	LR_{tr}	p-value
6	2	.0493	28.71	.00
	1	.0099	4.72	.32
8	2	.0409	25.63	.01
	1	.0125	5.92	.20
10	2	.0572	32.34	.00
	1	.0099	4.67	.32
12	2	.0575	30.96	.00
	1	.0069	3.24	.54

(B) *Testing for Stationarity*

	u_t		π_t	
# lags	LR	p-value	LR	p-value
6	15.04	.00	6.48	.01
8	8.56	.00	7.35	.01
10	13.22	.00	14.07	.00
12	15.24	.00	14.99	.00

(C) *Estimates of* $\pi_t - \beta_u u_t$

# lags	β_u	$\ln L$	AIC	BIC	LIL
6	1.22	−943.58	4.10	4.74	4.20
8	1.71	−924.50	4.07	4.88	4.20
10	1.91	−907.34	4.05	5.05	4.21
12	1.84	−889.93	4.03	5.20	4.21

TABLE 4: Testing for serial correlation and ARCH for the U. S. in a linear VAR(k) model, 1959:1–1998:12

(A) *Serial Correlation Tests*

# lags	# Unit Roots	Ljung-Box Test	p-value	LM Test	p-value
6	0	527.32	.01	18.81	.00
	1	526.18	.01	18.60	.00
8	0	481.44	.08	19.48	.00
	1	482.53	.09	18.75	.00
10	0	485.36	.03	20.84	.00
	1	486.99	.03	20.09	.00
12	0	460.10	.09	12.02	.02
	1	459.91	.10	11.72	.02

NOTES: The Ljung-Box test concerns the first 118 autocorrelations, while the LM statistic concerns serial correlation at the 12th lag for the residuals.

(B) *Testing for ARCH*

		u_t-equation		π_t-equation	
# lags	# Unit Roots	ARCH(k)	p-value	ARCH(k)	p-value
6	0	20.49	.00	44.35	.00
	1	20.62	.00	42.51	.00
8	0	16.48	.04	33.47	.00
	1	16.30	.04	32.64	.00
10	0	22.04	.01	26.43	.00
	1	21.06	.02	26.09	.00
12	0	17.82	.12	25.32	.01
	1	18.60	.10	25.48	.01

TABLE 5: Specification based on conditional scores in 2-state MS-VAR(k) systems for the U. S., 1959:1–1998:12

(A) *Equation-by-equation Tests*

Hypothesis	System 1 ($k = 2$) $\pi_t - .039u_t$	System 1 ($k = 2$) Δu_t	System 2 ($k = 2$) π_t	System 2 ($k = 2$) Δu_t	System 3 ($k = 3$) π_t	System 3 ($k = 3$) u_t
Autocorrelation	.74	.61	.78	.62	.71	1.39
p-value	.56	.65	.54	.65	.59	.24
ARCH	.80	1.22	.84	1.12	1.34	.46
p-value	.53	.34	.50	.35	.25	.76
Markov	.25	.38	.26	.37	.27	1.78
p-value	.91	.82	.91	.83	.89	.13

(B) *System Tests*

Hypothesis	System 1 ($\beta_u = 0.039$)	System 2 ($\beta_u = 0$)	System 3 (π_t, u_t)
Autocorrelation	.63	.64	.92
p-value	.86	.86	.55
ARCH	.96	.95	.79
p-value	.54	.56	.81
Markov	.30	.31	1.35
p-value	.94	.93	.23

(C) *System Properties*

	System 1 ($\beta_u = 0.039$)	System 2 ($\beta_u = 0$)	System 3 (π_t, u_t)
$\ln L(\hat{\theta})$	−922.63	−922.65	−910.01
AIC	3.98	3.98	3.97
BIC	4.58	4.58	4.66
LIL	4.07	4.07	4.08
$\hat{e}_1$	.65	.65	.96
$\hat{\varpi}_1$	.42	.42	.45
$\hat{\sigma}_{\varpi_1}$	.15	.15	.16

TABLE 6: Estimated unconditional and conditional means and covariances for inflation and unemployment in the U. S., 1959:1–1998:12

(A) *Unconditional Moments*

System	Variable	Mean	Variance	Covariance
1	$\pi_t - .039u_t$	3.85 (.60)	12.38 (3.44)	.06 (.06)
	Δu_t	−.01 (.02)	.04 (.01)	
2	π_t	4.09 (.60)	12.39 (3.44)	.06 (.06)
	Δu_t	−.01 (.02)	.04 (.01)	

(B) *Conditional Moments*

System	Variable	Mean	Variance	Covariance
Regime 1				
1	$\pi_t - .039u_t$	5.31 (1.01)	20.74 (4.48)	.05 (.13)
	Δu_t	.02 (.08)	.06 (.01)	
2	π_t	5.58 (1.01)	20.65 (4.47)	.05 (.13)
	Δu_t	.02 (.03)	.06 (.01)	
Regime 2				
1	$\pi_t - .039u_t$	2.80 (.20)	3.72 (.43)	.00 (.02)
	Δu_t	−.03 (.01)	.02 (.00)	
2	π_t	3.02 (.20)	3.73 (.43)	.00 (.02)
	Δu_t	−.03 (.01)	.02 (.00)	

TABLE 7: Cointegration analysis for bivariate VAR(k) models of inflation and unemployment for Sweden (1970:2–1998:12) and the U. K. (1950:4–1998:12)

(A) *Cointegration Tests*

		Sweden			U. K.		
# lags	# Unit Roots	Eigenvalue	LR_{tr}	p-value	Eigenvalue	LR_{tr}	p-value
6	2	.1312	50.00	.00	.0426	27.59	.00
	1	.0060	2.04	.77	.0041	2.39	.70
8	2	.0687	26.25	.01	.0372	24.54	.01
	1	.0063	2.14	.75	.0046	2.68	.64
10	2	.0573	21.97	.03	.0396	26.10	.01
	1	.0062	2.08	.76	.0050	2.86	.61
12	2	.0370	14.61	.25	.0354	23.11	.02
	1	.0059	1.97	.78	.0042	2.44	.69

(B) *Testing for Stationarity*

	Sweden				U. K.			
	u_t		π_t		u_t		π_t	
# lags	LR	p-value	LR	p-value	LR	p-value	LR	p-value
6	45.90	.00	26.35	.00	22.46	.00	.18	.67
8	21.95	.00	13.73	.00	18.54	.00	.48	.49
10	17.75	.00	11.98	.00	19.20	.00	1.09	.30
12	10.55	.00	7.77	.01	15.67	.00	2.69	.10

(C) *Estimates of* $\pi_t - \beta_u u_t$

	Sweden					U. K.				
# lags	β_u	$\ln L$	AIC	BIC	LIL	β_u	$\ln L$	AIC	BIC	LIL
6	−1.24	−1112.82	6.70	7.52	6.83	.11	−1561.86	5.50	6.03	5.58
8	−1.24	−1099.26	6.70	7.75	6.86	.18	−1538.75	5.46	6.15	5.57
10	−1.26	−1089.06	6.73	8.02	6.93	.26	−1524.48	5.46	6.30	5.59
12	−1.32	−1067.76	6.69	8.21	6.93	.45	−1497.78	5.41	6.40	5.57

TABLE 8: System properties for the MS-VAR(k) models using Swedish (1970:2–1998:12) and U. K. (1950:4–1998:12) data on inflation and unemployment.

(A) *Sweden*

	System 1 $\beta_u = -0.957$, $(k = 2)$	System 2 $\beta_u = 0$, $(k = 2)$	System 3 (π_t, u_t), $(k = 2)$
$\ln L(\hat{\theta})$	-1100.96	-1120.08	-1095.66
AIC	6.54	6.66	6.51
BIC	7.33	7.44	7.30
LIL	6.67	6.78	6.64
$\hat{e}_1$	.28	.26	1.02
$\hat{\varpi}_1$	.39	.43	-
$\hat{\sigma}_{\varpi_1}$	.08	.08	-

(B) *U. K.*

	System 1 $\beta_u = .008$, $(k = 2)$	System 2 $\beta_u = 0$, $(k = 2)$	System 3 (π_t, u_t), $(k = 3)$
$\ln L(\hat{\theta})$	-1500.87	-1500.87	-1478.80
AIC	5.25	5.25	5.21
BIC	5.77	5.77	5.87
LIL	5.34	5.34	5.31
$\hat{e}_1$	.57	.57	.98
$\hat{\varpi}_1$	.62	.62	.73
$\hat{\sigma}_{\varpi_1}$	.08	.08	.27

Table 9: Estimated unconditional and conditional means and covariances for inflation and unemployment in Sweden (1970:2–1998:12) and the U. K. (1950:4–1998:12)

	Sweden			U. K.		
Variable	Mean	Variance	Covariance	Mean	Variance	Covariance
Unconditional moments						
π_t	6.53 (.63)	51.36 (6.99)	.11 (.08)	6.57 (.85)	55.19 (6.88)	.19 (.07)
Δu_t	.01 (.01)	.07 (.01)		.01 (.01)	.03 (.01)	
Regime 1 moments						
π_t	8.52 (1.12)	88.26 (13.98)	.08 (.18)	7.52 (1.25)	79.44 (7.38)	.29 (.08)
Δu_t	.04 (.03)	.05 (.01)		−.00 (.01)	.01 (.00)	
Regime 2 moments						
π_t	5.05 (.56)	18.37 (3.11)	.06 (.07)	5.06 (.42)	12.43 (1.67)	.06 (.07)
Δu_t	−.01 (.02)	.08 (.01)		.02 (.03)	.06 (.01)	

FIGURE 1: Inflation and unemployment series for the U. S. in levels and first differences, 1959:1–1998:12. The shaded areas are the maximum posterior estimates of the Regime 2 periods for System 2.

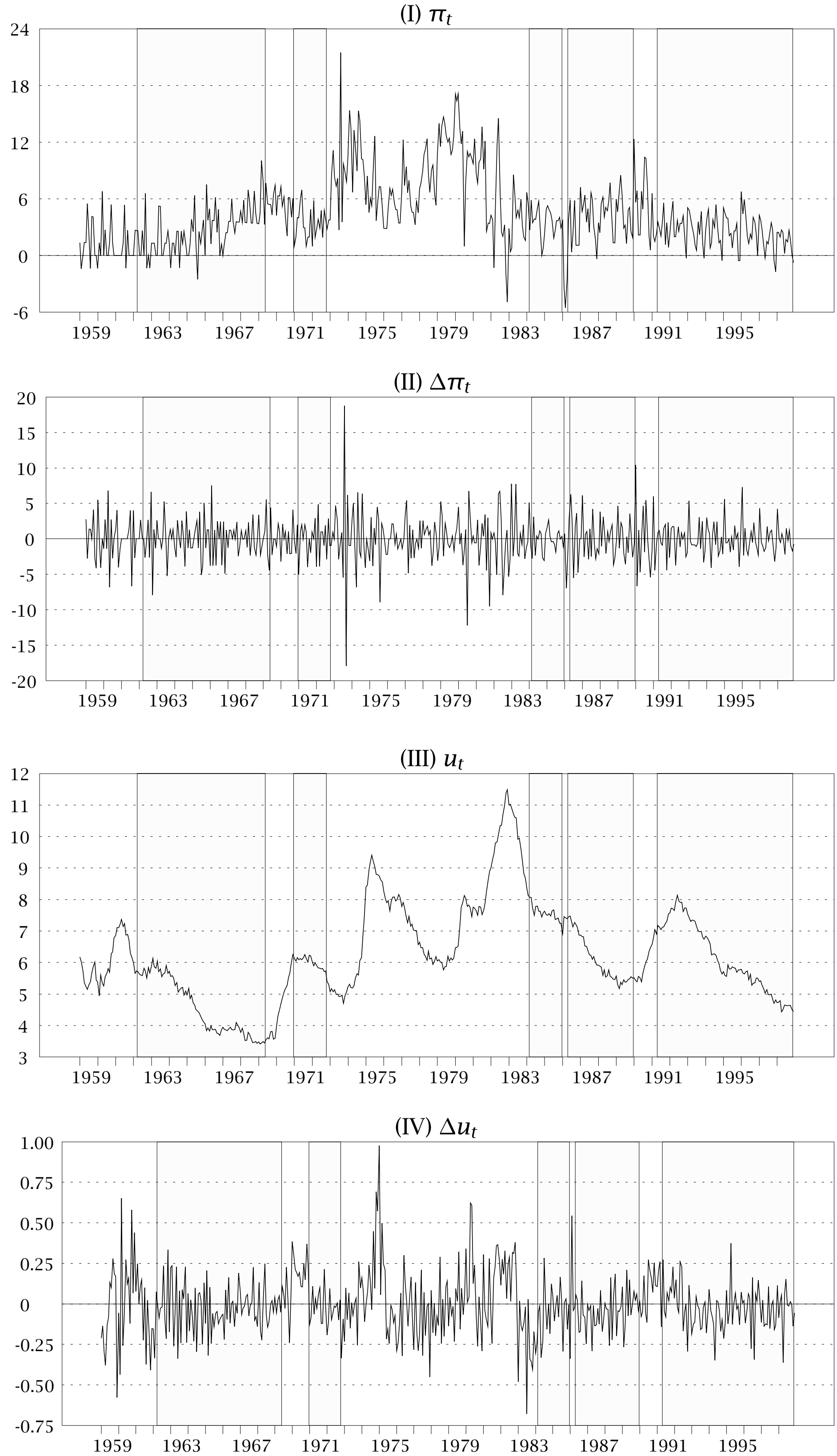
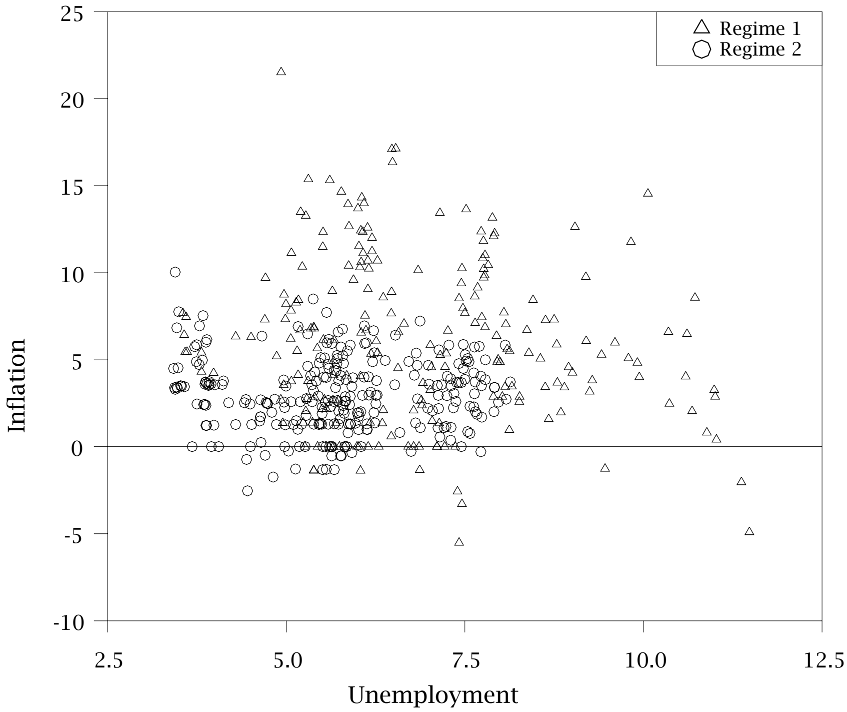


FIGURE 2: Unemployment and inflation in the U. S. for the sample 1959:1–1998:12

(I) *Monthly inflation*



(II) *Yearly inflation*

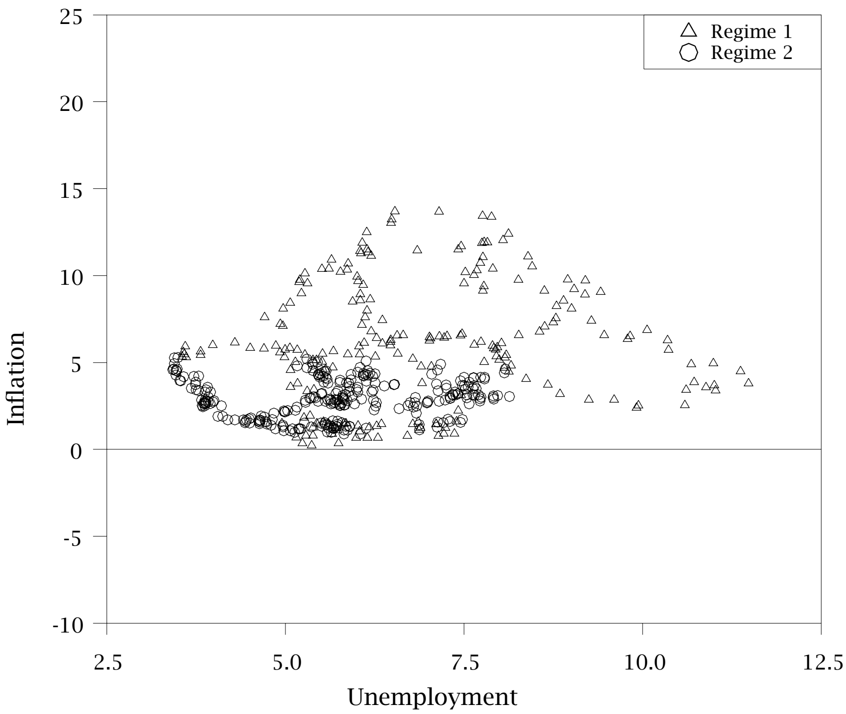


FIGURE 3: The scaled log-likelihood function (solid line) and the estimated maximum eigenvalue (dashed line) for 2-state MS-VAR(2) systems for the U. S., 1959:1–1998:12

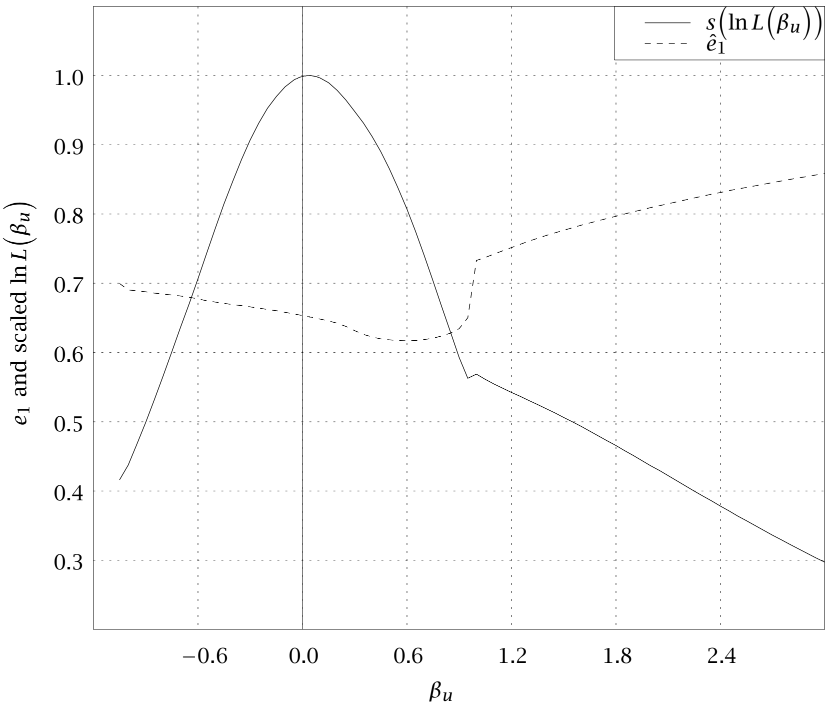


FIGURE 4: Estimated smooth probabilities of being in Regime 1 for 2-state MS-VAR(k) models for the U. S., 1959:1–1998:12

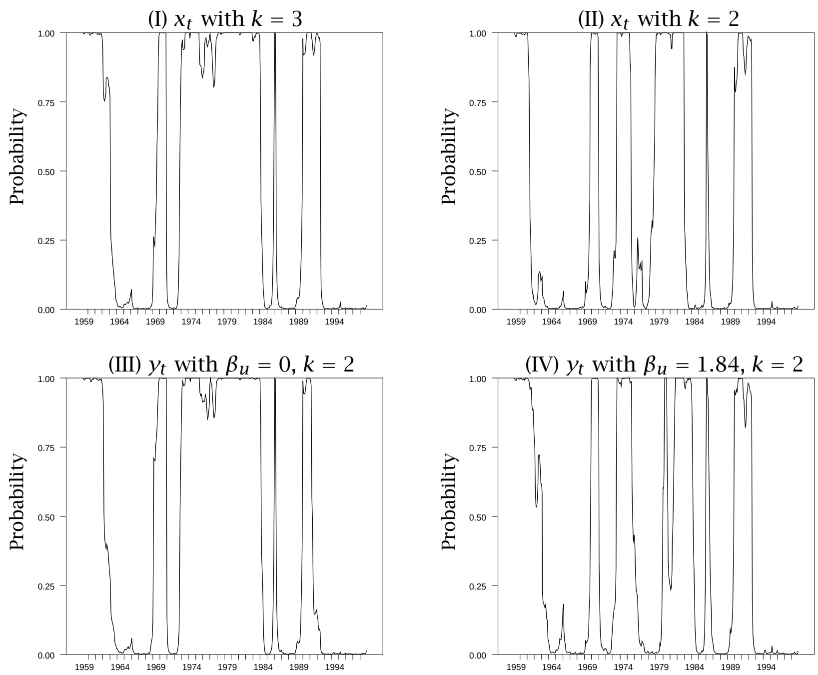
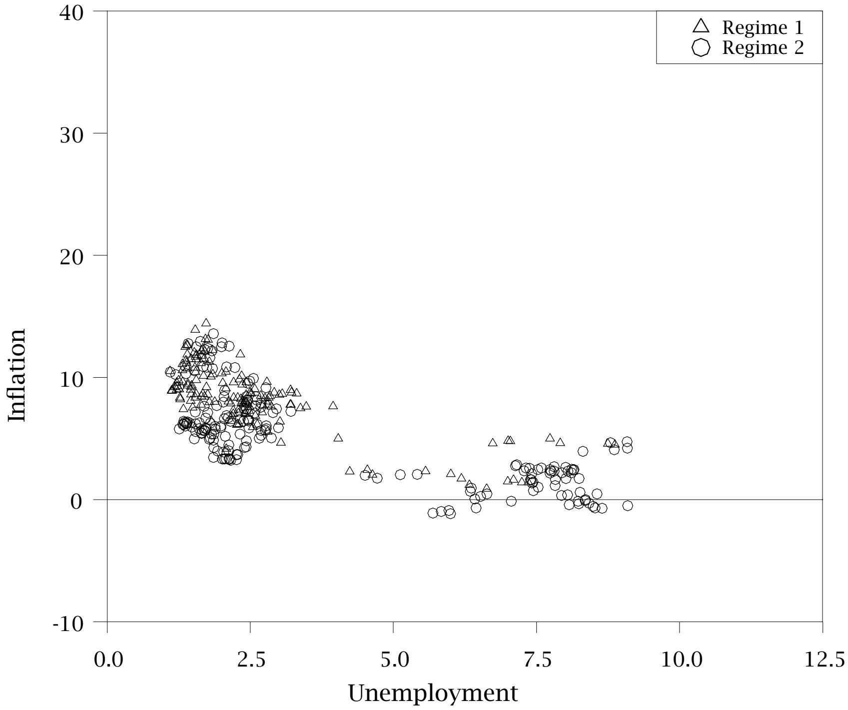


FIGURE 5: Unemployment and yearly inflation in Sweden and the U. K.

(I) *Sweden*



(II) *U. K.*

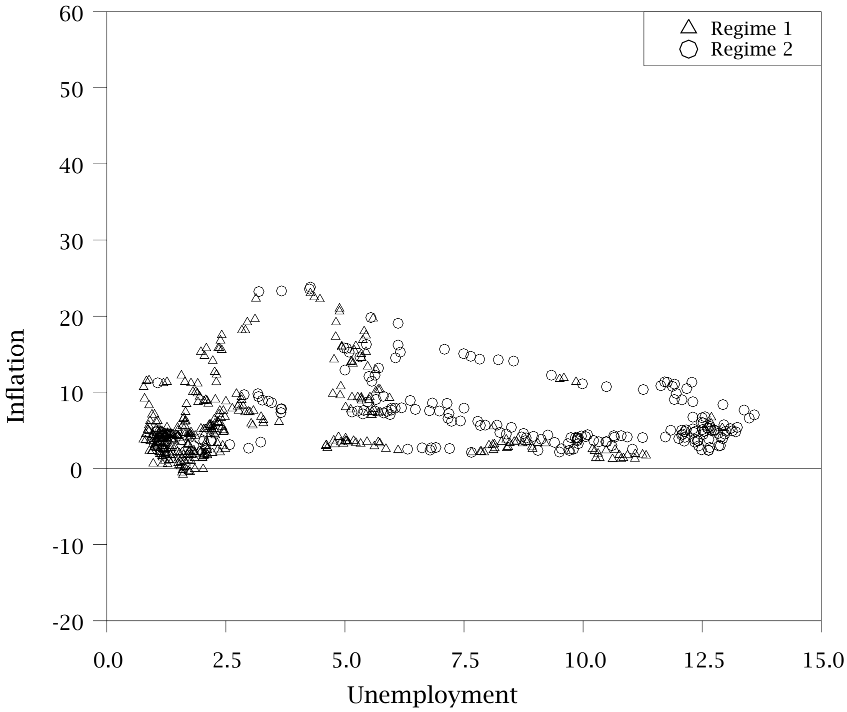
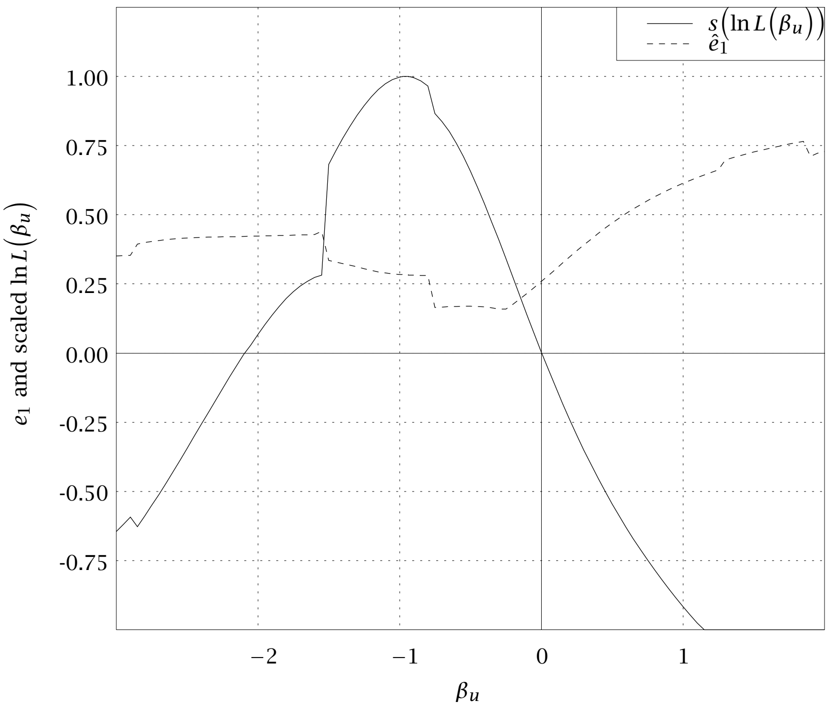


FIGURE 6: The scaled log-likelihood function (solid line) and the estimated maximum eigenvalue (dashed line) for 2-state MS-VAR(2) systems for Sweden (1970:2–1998:12) and the U. K. (1950:4–1998:12)

(I) *Sweden*



(II) *U. K.*

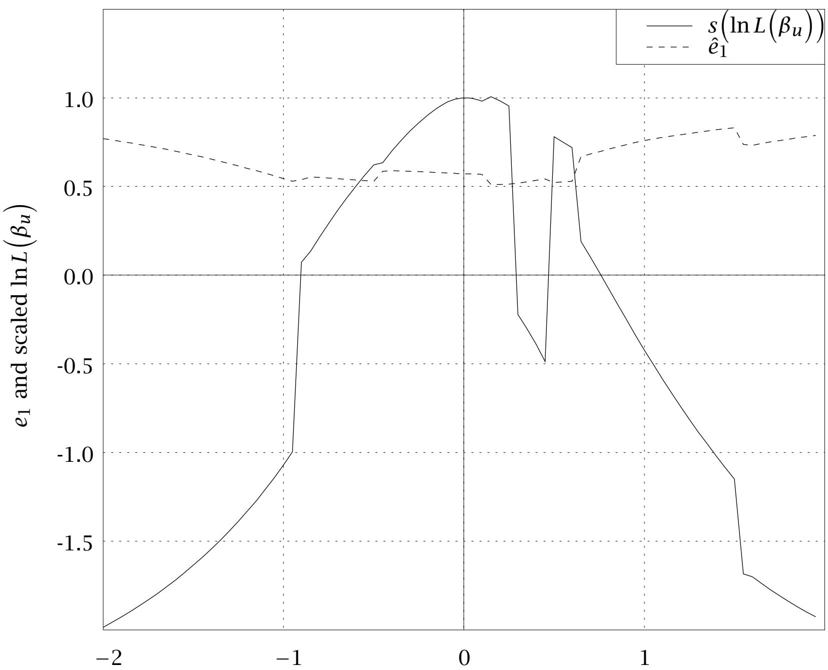
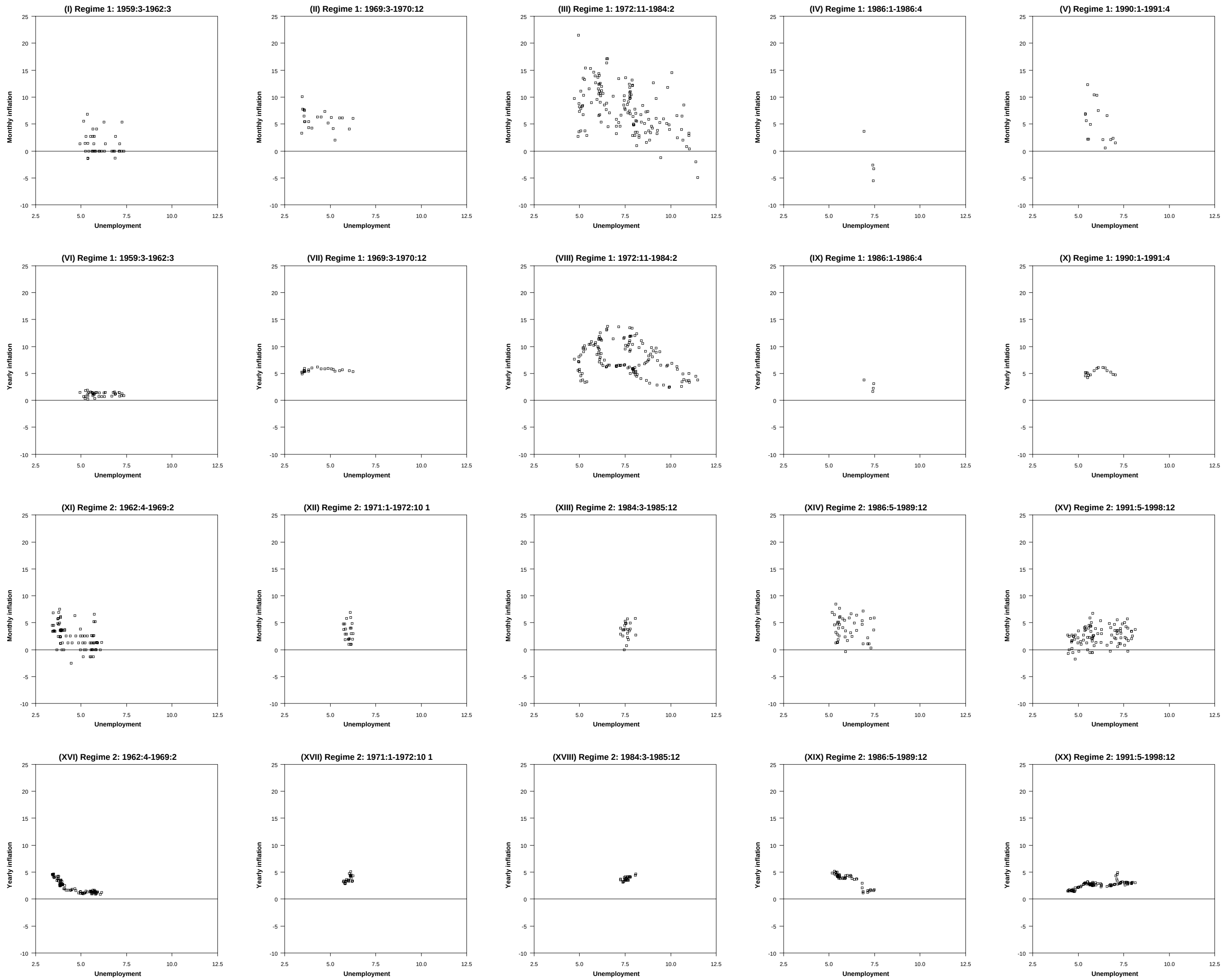


Figure 7: Unemployment and monthly/yearly inflation in the U. S. for the maximum posterior estimates of the Regime 1 and Regime 2 periods using System 2.



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